

Justifying Inference Rule: OR_L

$$\frac{H, P \vdash R \quad H, Q \vdash R}{H, P \vee Q \vdash R} \text{ OR_L}$$

Example Inference Rules

$$\frac{}{\vdash 0 \in \mathbb{N}} \quad \mathbf{P1}$$

$$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}} \quad \mathbf{P2}$$

$$\frac{}{0 < n \vdash n-1 \in \mathbb{N}} \quad \mathbf{P2'}$$

$$\frac{}{n \in \mathbb{N} \vdash 0 \leq n} \quad \mathbf{P3}$$

$$\frac{}{n < m \vdash n+1 \leq m} \quad \mathbf{INC}$$

$$\frac{}{n \leq m \vdash n-1 < m} \quad \mathbf{DEC}$$

$$\frac{H, P \vdash R \quad H, Q \vdash R}{H, P \vee Q \vdash R} \quad \mathbf{OR_L}$$

$$\frac{H \vdash P}{H \vdash P \vee Q} \quad \mathbf{OR_R1}$$

$$\frac{H \vdash Q}{H \vdash P \vee Q} \quad \mathbf{OR_R2}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \quad \mathbf{MON}$$

Discharging **PO**s of original m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $\vdash$
 $n + 1 \in \mathbb{N}$

ML_in/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $\vdash$
 $n - 1 \in \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $\vdash$
 $n + 1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $\vdash$
 $n - 1 \leq d$

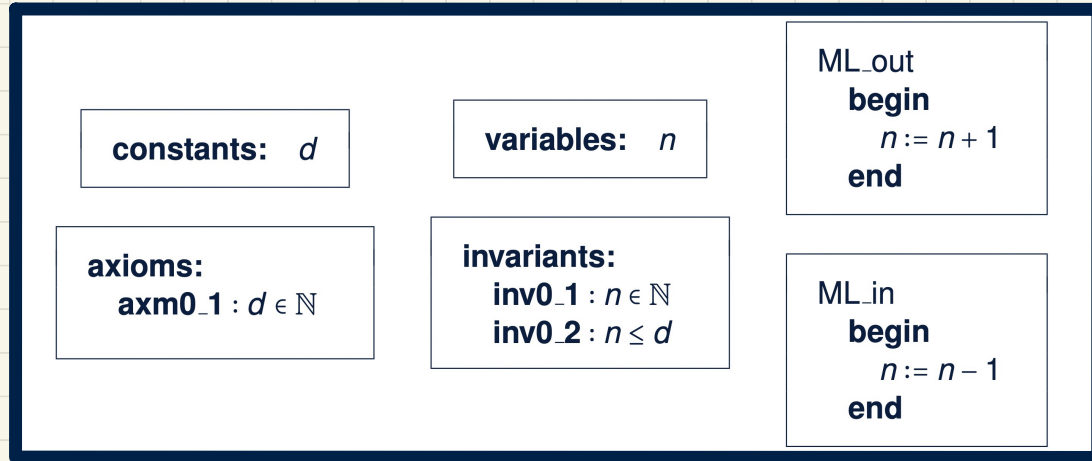
$$\frac{H \vdash P}{H \vdash P \vee Q} \text{ OR.R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \text{ MON}$$

$$\frac{}{n \leq m \vdash n - 1 < m} \text{ DEC}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}} \text{ P2}$$

PO/VC Rule of **Invariant** Preservation: Revised M0



$A(c)$
 $I(c, \mathbf{v})$
 $G(c, \mathbf{v})$
 $\vdash$
 $I_i(c, \mathbf{E}(c, \mathbf{v}))$

Q. How many PO/VC rules for model m0?

Discharging **PO**s of revised m0: Invariant Preservation

ML_out/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n < d$
 $\vdash$
 $n + 1 \in \mathbb{N}$

ML_in/inv0_1/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n > 0$
 $\vdash$
 $n - 1 \in \mathbb{N}$

ML_out/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n < d$
 $\vdash$
 $n + 1 \leq d$

ML_in/inv0_2/INV

$d \in \mathbb{N}$
 $n \in \mathbb{N}$
 $n \leq d$
 $n > 0$
 $\vdash$
 $n - 1 \leq d$

$$\frac{H \vdash P}{H \vdash P \vee Q} \text{ OR_R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \text{ MON}$$

$$\frac{}{n \leq m \vdash n - 1 < m} \text{ DEC}$$

$$\frac{}{n < m \vdash n + 1 \leq m} \text{ INC}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}} \text{ P2}$$

$$\frac{}{0 < n \vdash n - 1 \in \mathbb{N}} \text{ P2'}$$